Imperial College London

Travel Time in Uncertain Environments

Presentation made in honour of
Professor Enrico Magenes
Pavia 22-07-2010

Erol Gelenbe

http://www.ee.ic.ac.uk/gelenbe

Professor in the Dennis Gabor Chair

Head of Intelligent Systems and Networks

Dept of Electrical and Electronic Engineering

Imperial College

London SW7 2BT

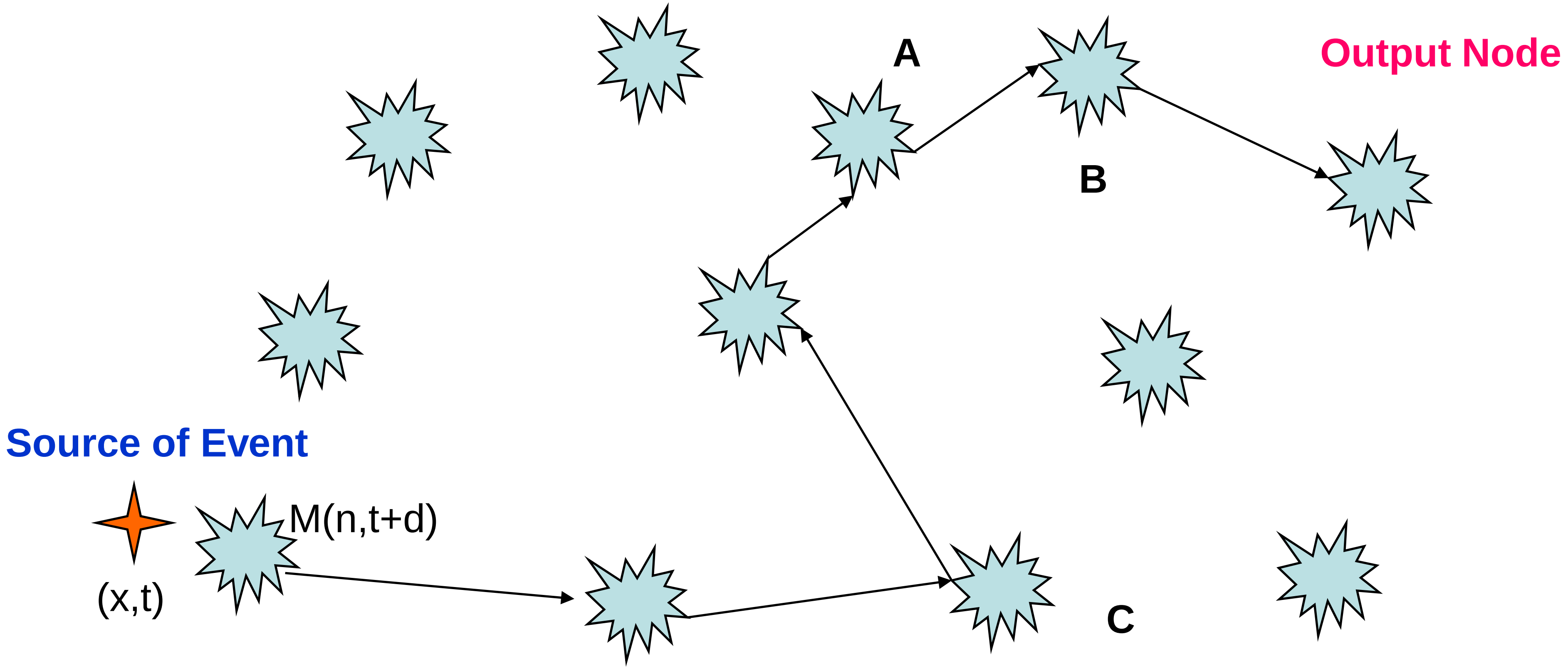
http://san.ee.ic.ac.uk

Motivation

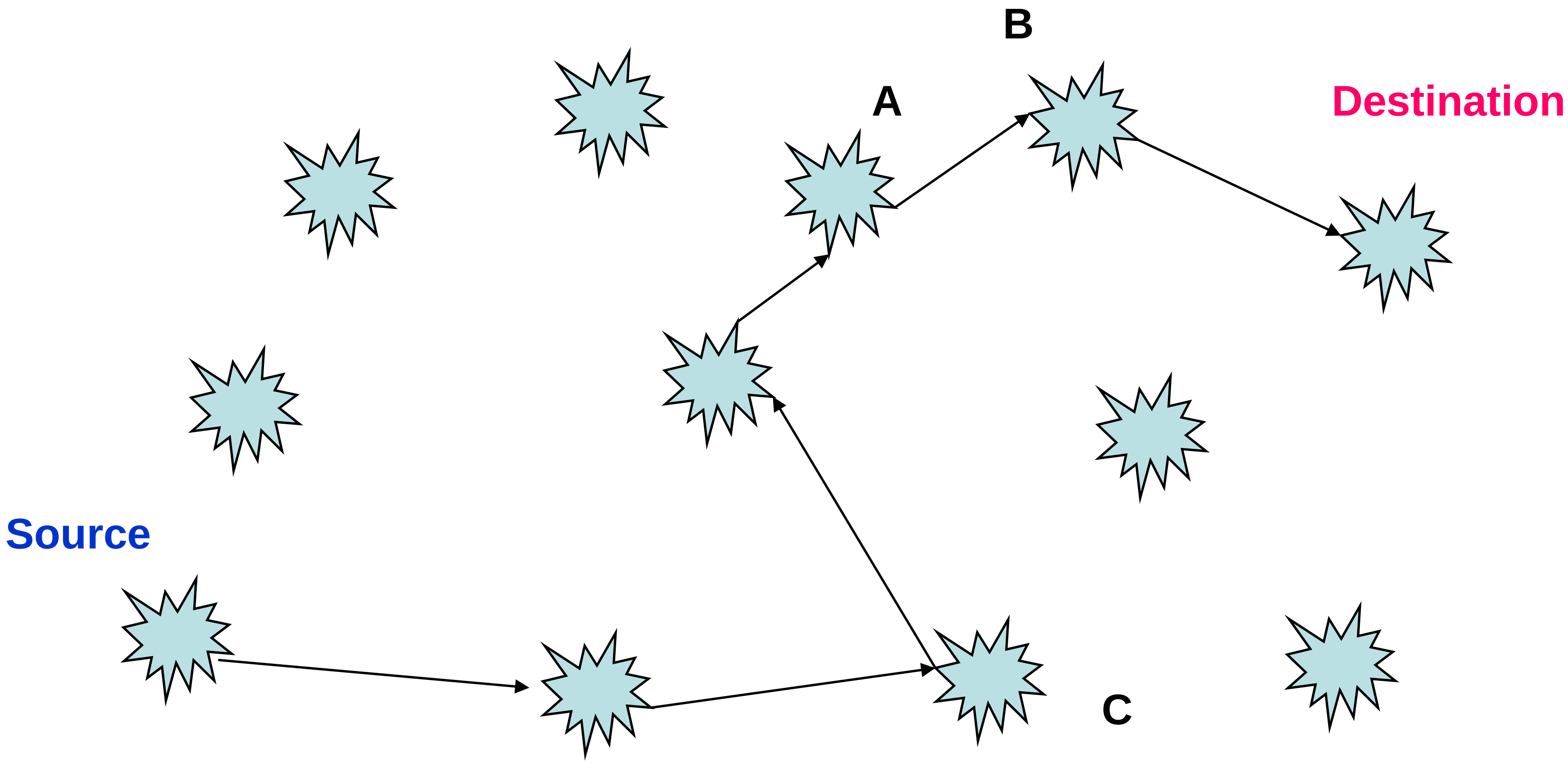
- Robotic or Military Search in an Unknown Universe
- Packets Travelling in an Unknown Network esp. Wireless
- Search by Software Robots for Data in a Very Large Distributed Database
- Biological Agents Diffusing through a Random Medium until they Encounter a Docking Point
- Particles Moving in a Random Medium until they Encounter an Oppositely Charged Receptor
- Computation (e.g. Simulated Annealing) in Search for a Local Minimum on one or More Parallel Processors

Example from Wireless Sensor Networks

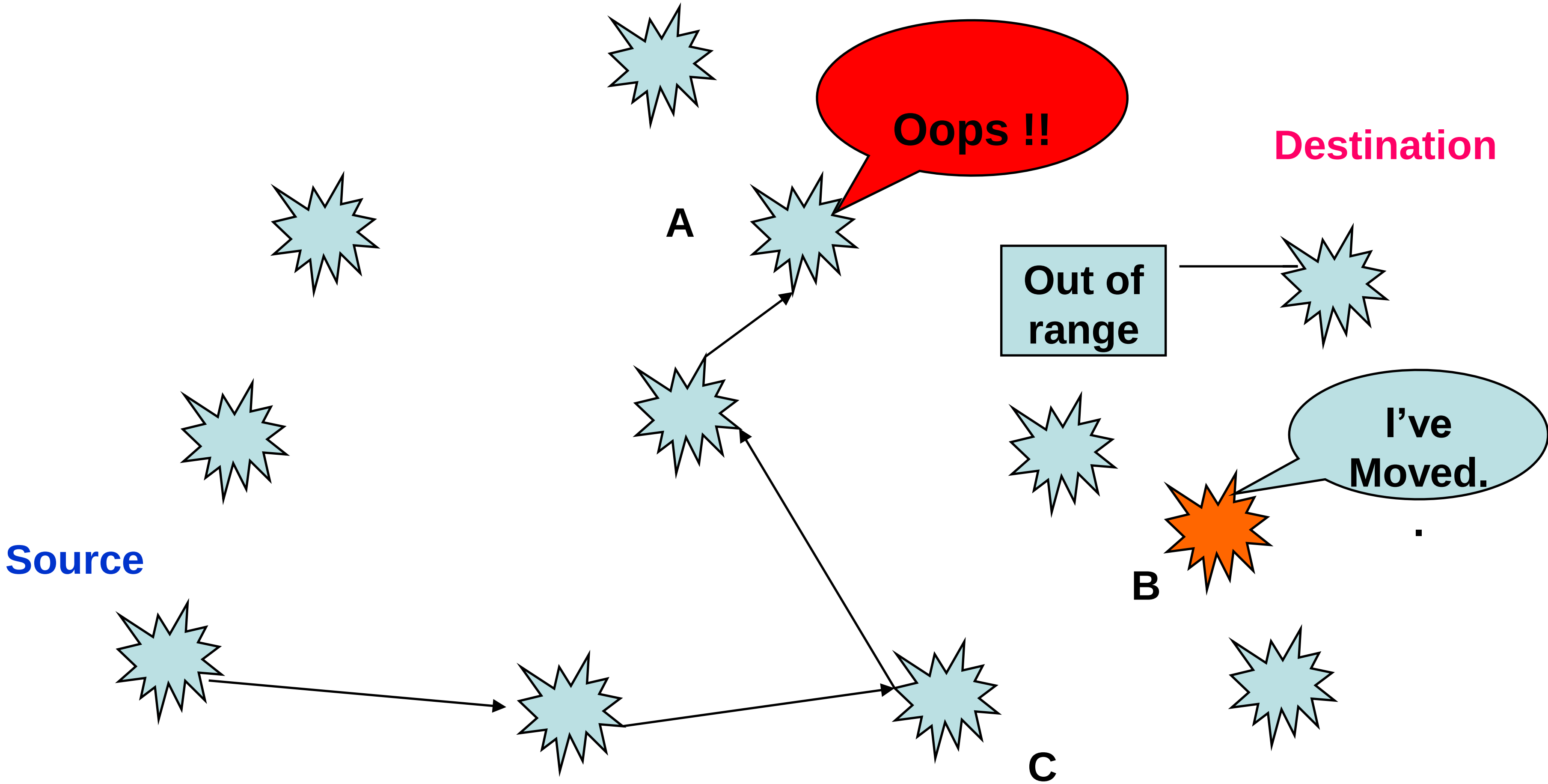
Event occuring at location (x,t) is reported by the Sensor Node at location (n,t+d) if $||X(n)-x||<\varepsilon$. The node sends out a packet at t+d. The packet containing M(n,X(n),t+d) travels over multiple hops and reaches the Output Node at time t+d+T



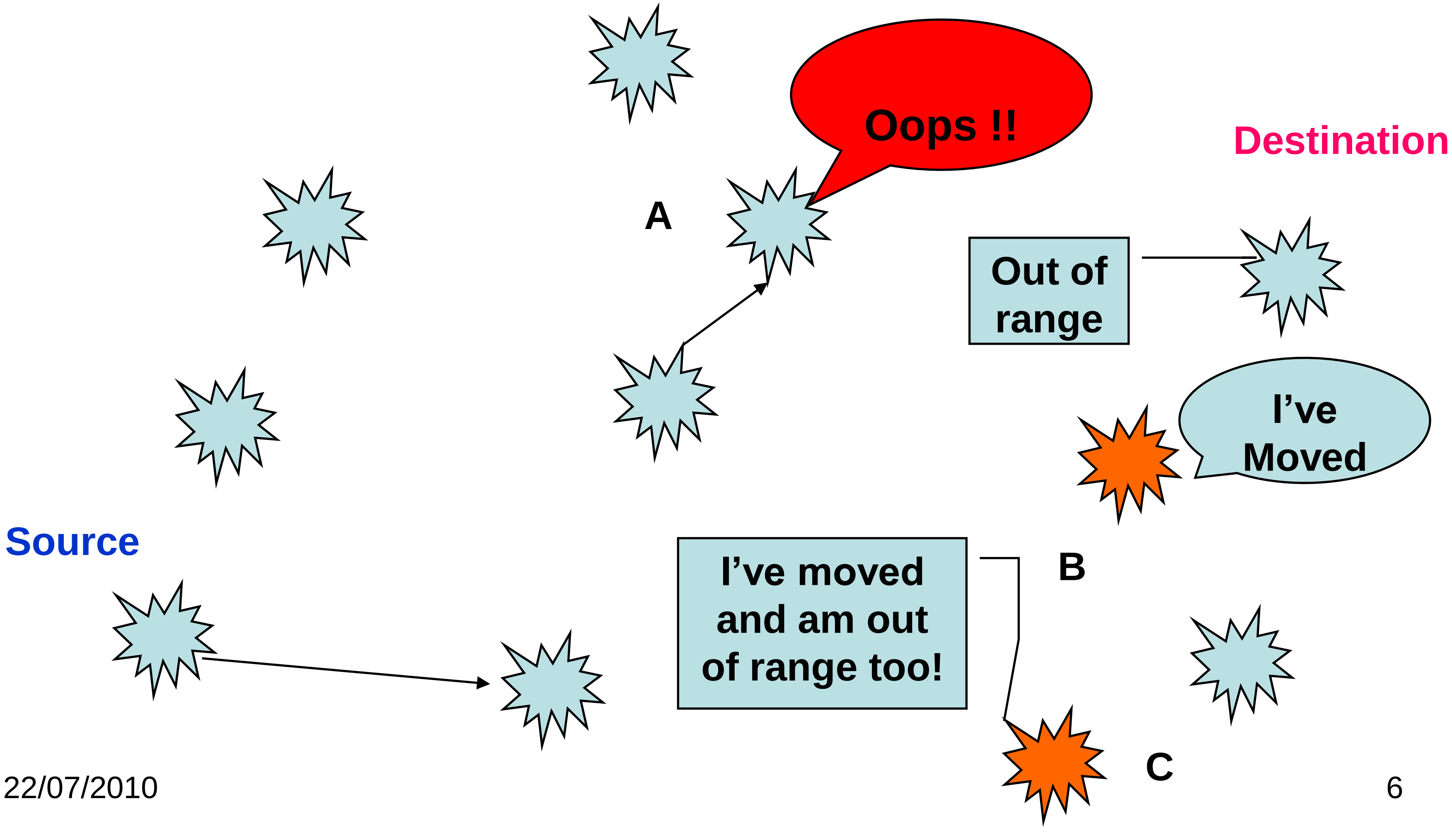
A Packet Needs to Go From S to Destination Using Multiple Hops .. But it is Ignorant about its Path and all Kinds of Bad Things Can Happen .. Can it Still Succeed?



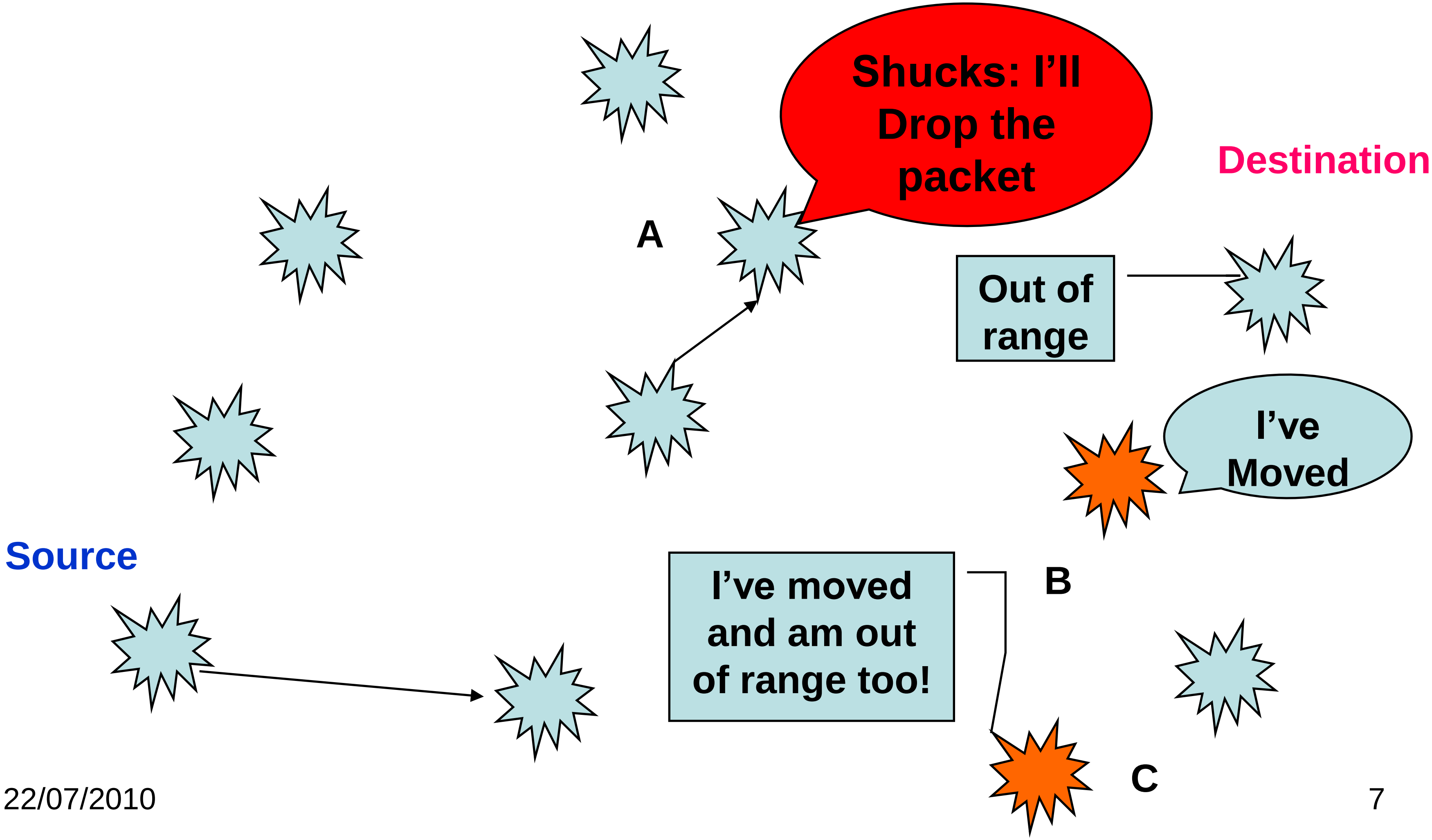
The packet may get to node A but there is no way to pursue the path towards the Destination



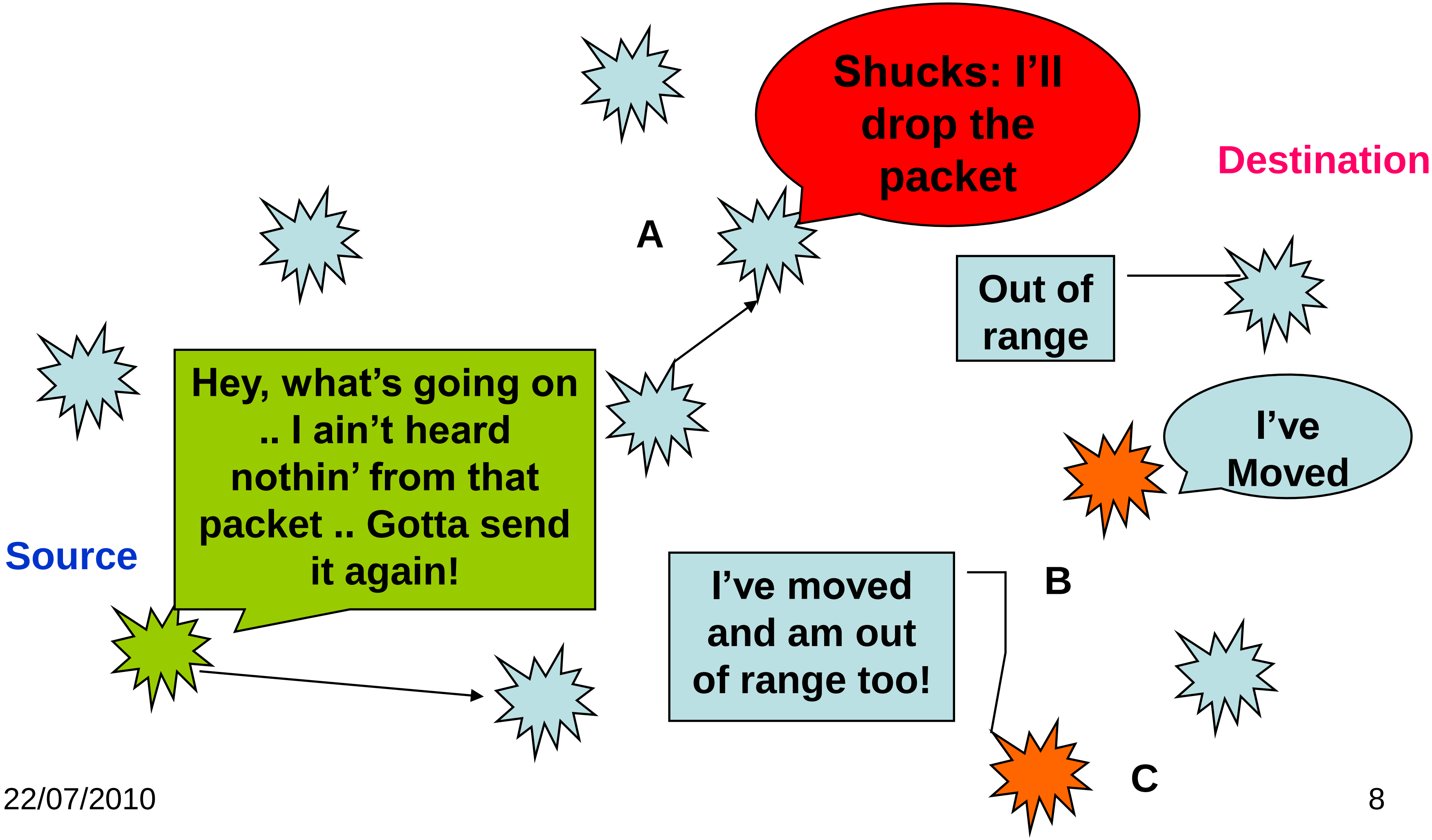
In the meanwhile the backward path to S breaks because C moves



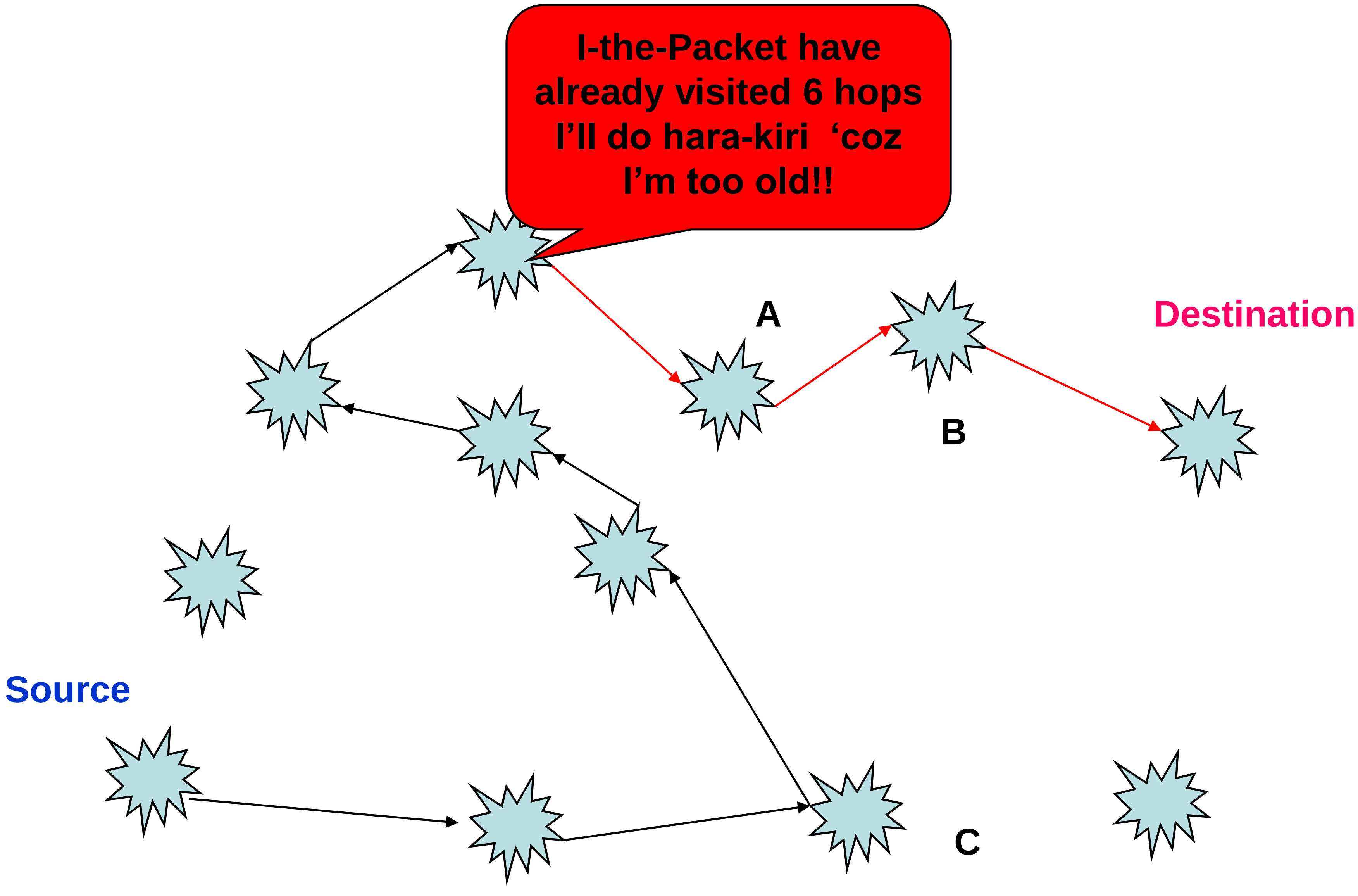
Now A Drops the Packet Because it Cannot Forward it



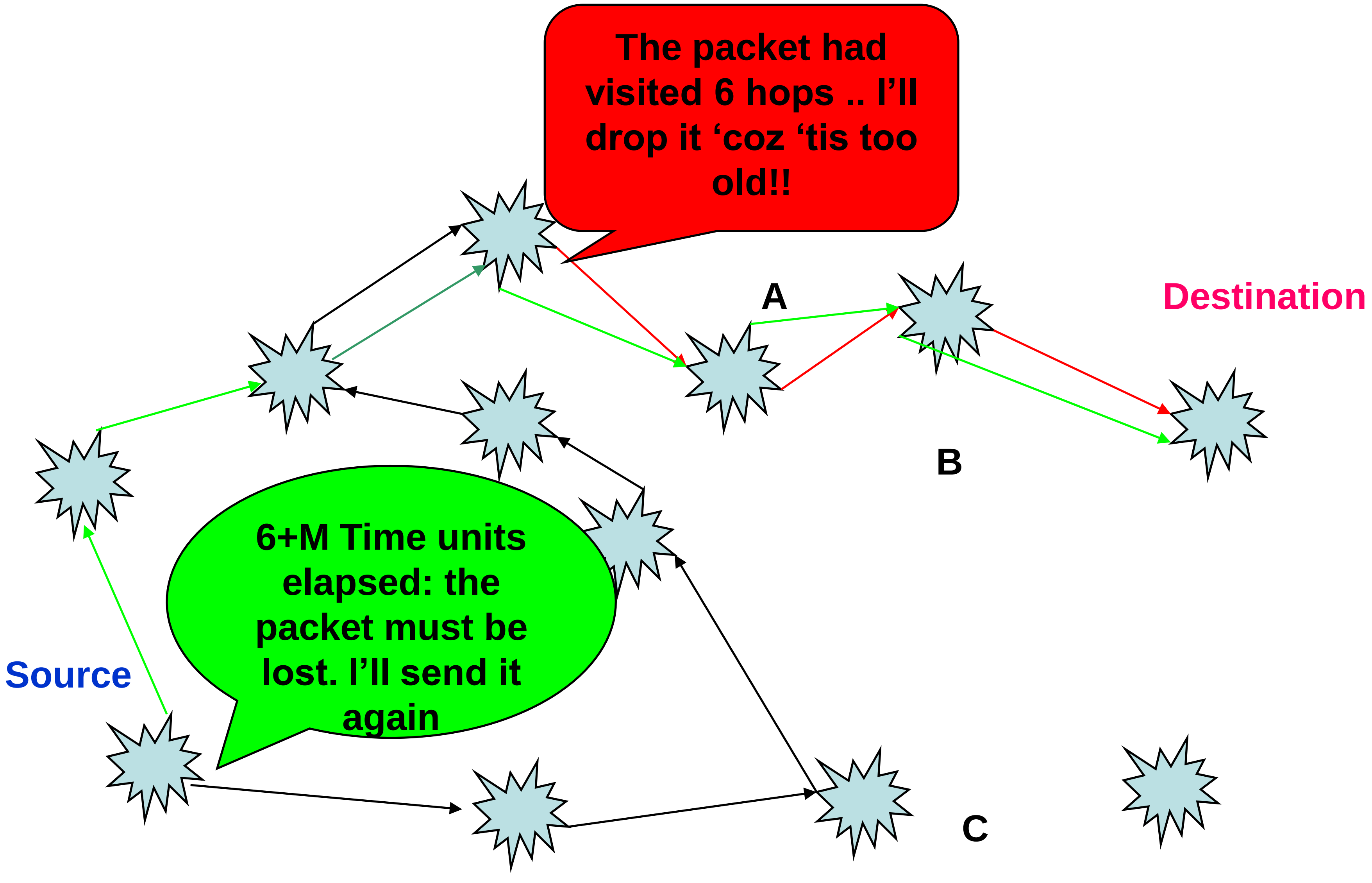
The backward path to S is broken, S has not received an ACK and it uses the time-out to retransmit the packet



Yet Another Situation .. Packet Hara Kiri



Some Time Later .. Packet Retransmission



General Context

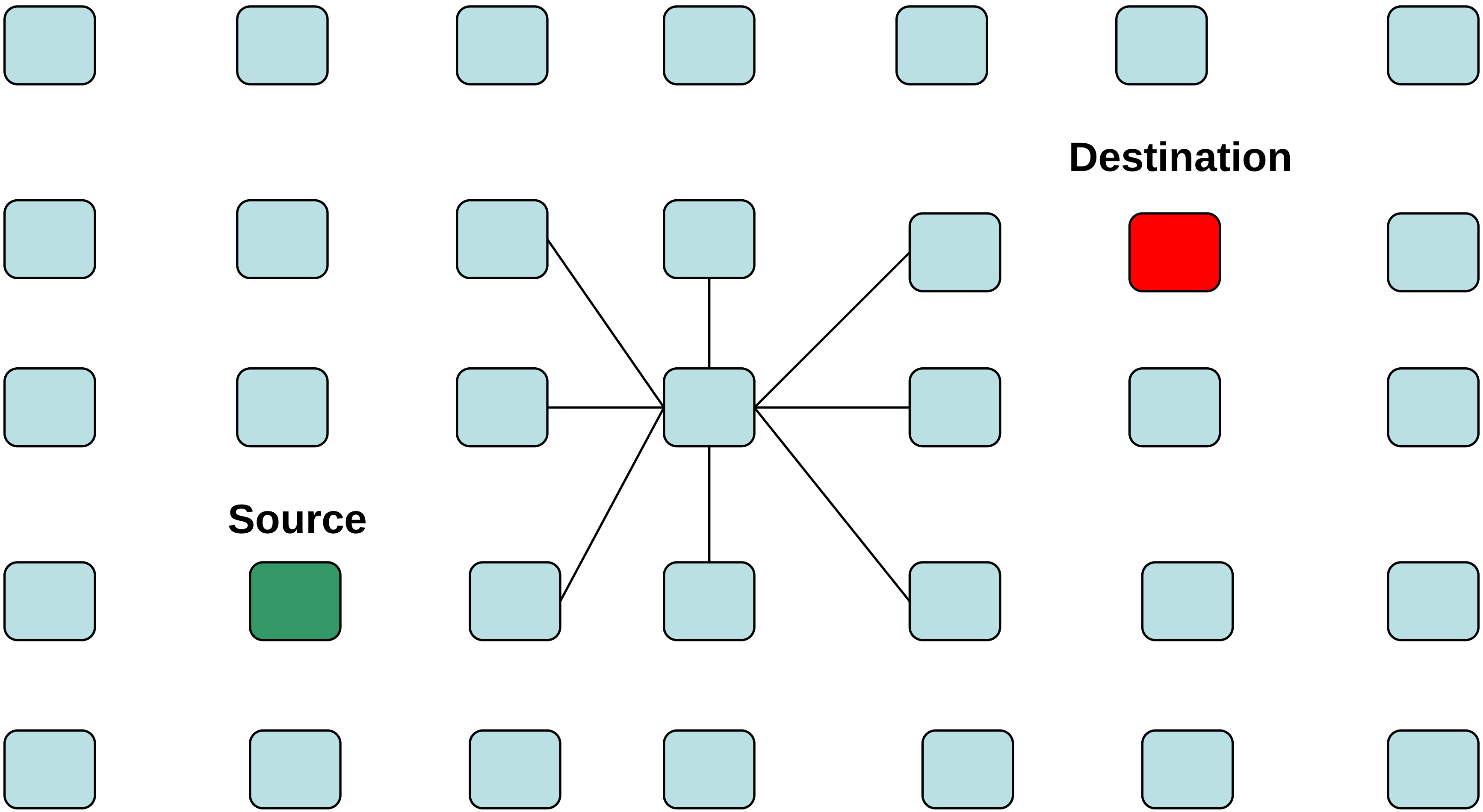
- Robotic searcher goes from some source S to find an object that is at distance D in some m-dimensional search space
- In a network, D is the destination distance in shortest number of hops
- In a time unit the searcher can move at most a distance of 1
- Conventional Wisdom: If you can estimate D and you have one searcher, then the exhaustive search will take time

$$\sim D^m$$

Network Context

- Packets go from some source S to a Destination (that may move) that is initially at distance D
- The wireless range is $\delta \ll D$, there are no collisions
- Packets can be lost in $[t,t+\Delta t]$ with probability $\lambda\Delta t$ anywhere on the path
- There is a time-out R (in time or number of hops), modelled as being timed-out in $[t,t+\Delta t]$ with probability $r\Delta t$ with a subsequent retransmission delay M
- Packets may or not know the direction they need to go – we do not nail down the routing scheme with any specific assumptions
- We avoid assumptions about the geography of nodes (in some m-dimensions) but assume temporal and spatial homogeneity and temporal and spatial independence along the distance to the destination – each physical setting will have a different distance ..

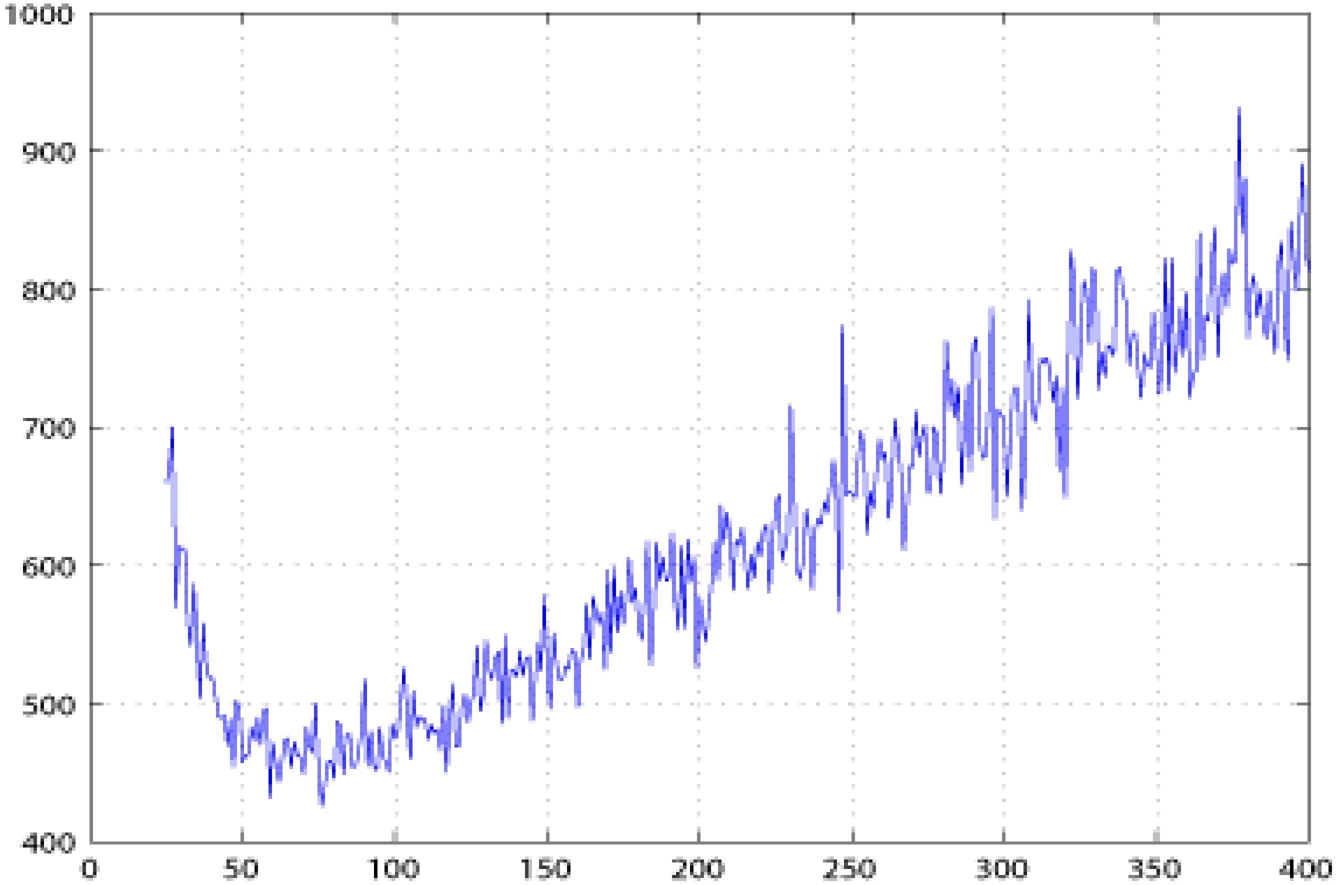
Simulation examples in an infinite grid: δ=1 (D=6) or δ=sqrt(2) (D=4)



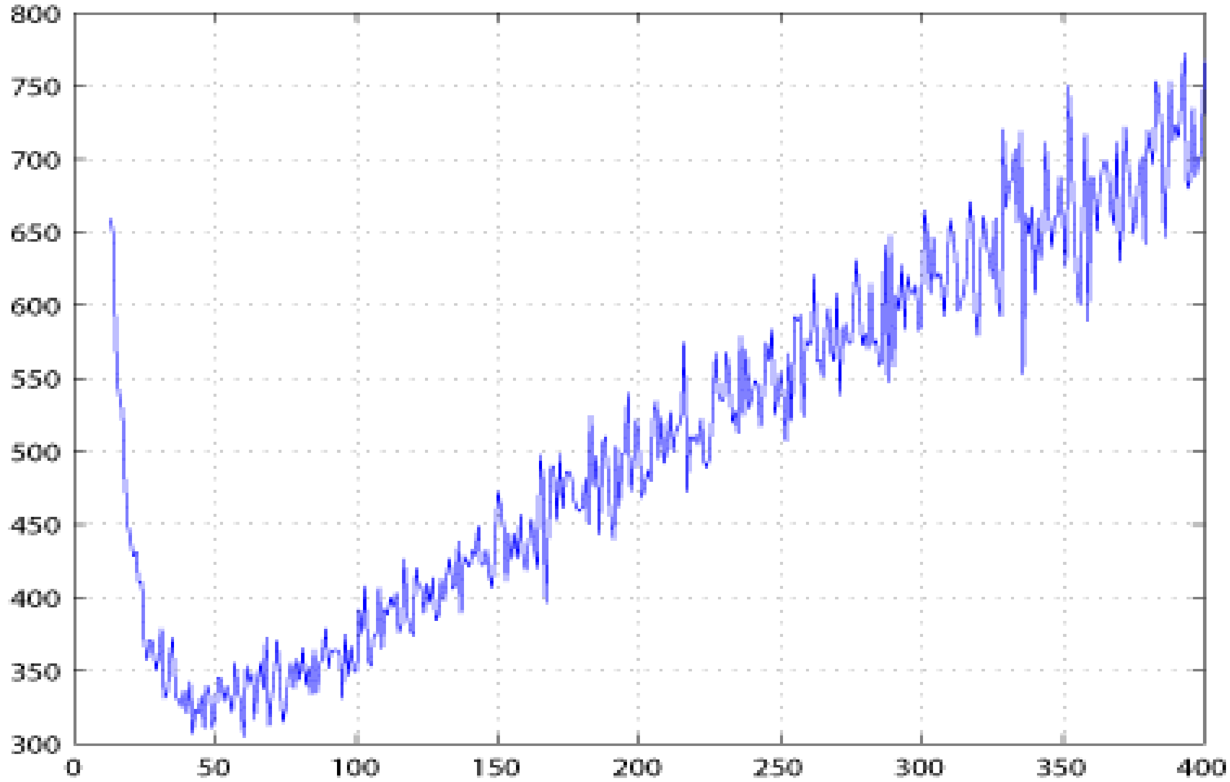
Simulations of Average Travel Time vs Constant Time-Out

δ=1, D=10, M=20, No Loss

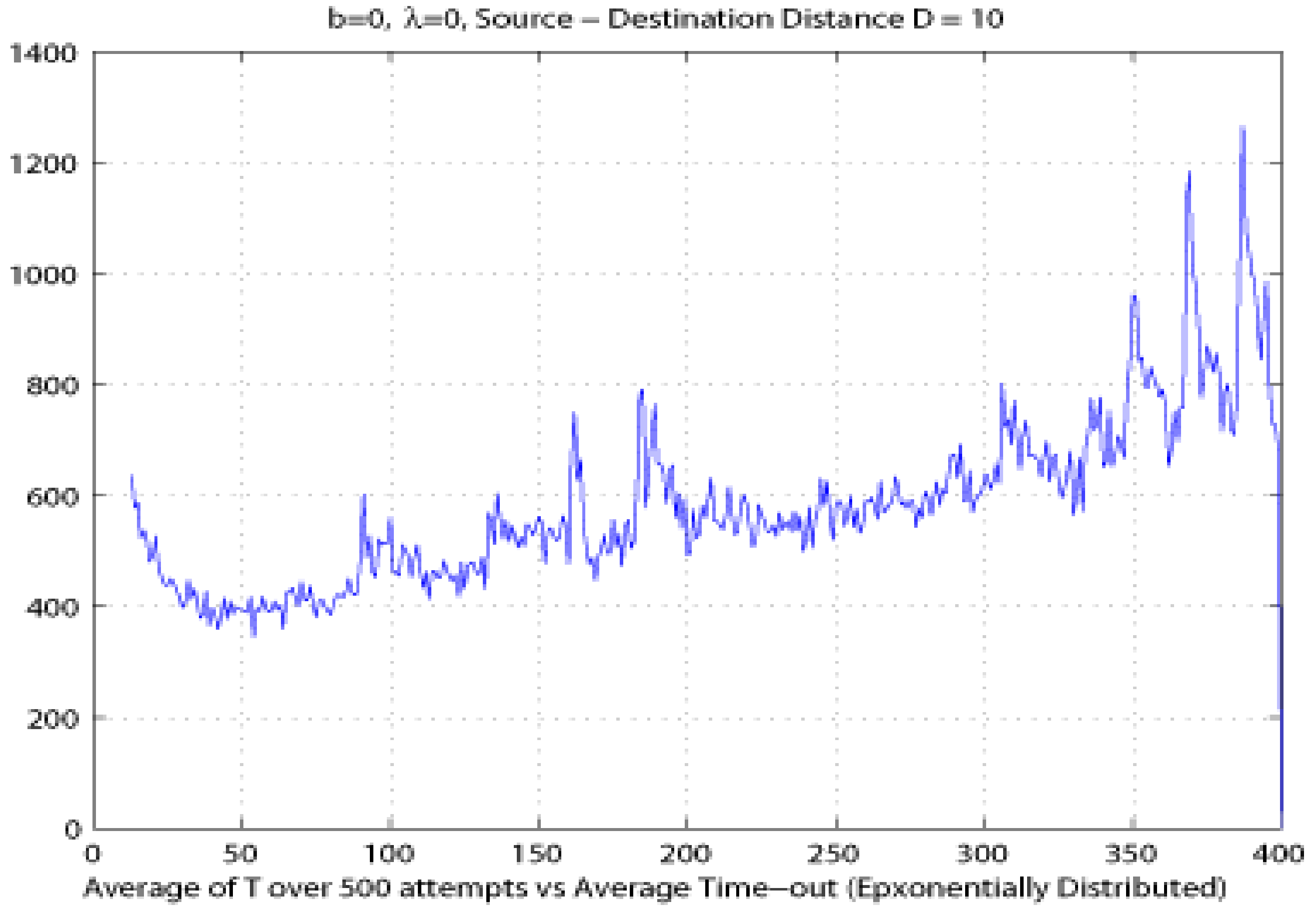
Perfect Ignorance: b=0, c=1



Average Travel Time vs Constant Time-Out
δ=sqrt(2), D=10, M=20, No Loss
Perfect Ignorance: b=0, c=1.5



Average Travel Time vs Avg of Exp Time-Out
D=10, M=20, 8-Neighbours, No Loss
Perfect Ignorance: b=0, c=1.5



Diffusion Model

- Do not consider the detailed topology of nodes,

- Assume homogeneity with respect to the distance to destination, and over time,

- Represent motion as a continuous process, for packets it would be a continuous approximation of discrete motion,

- Allow for loss (of packets) or destruction of the robotic searcher, or inactivation of the biological agent

- Include a time-out for the source to re-send the packet

- After each Time-Out, the sender waits M time units and then retransmits the packet under identical statistical conditions

Diffusion Model

- The distance of the searcher with respect to the destination at time t is X(t); it is homogeneous with respect to position and time

- Motion of the searcher is characterised by parameters b and c
    - The drift $b = E[X(t+\Delta t) - X(t)|X=x] / \Delta t$
    - The instantaneous variance

      $c = E[(X(t+\Delta t) - X(t)-b\Delta t)^2|X=x] /(\Delta t)^2$

- Loss (of packets), destruction of the robotic searcher, inactivation of the biological agent , represented by $\lambda\Delta t$

- Time-out is represented by $r\Delta t$, and after each Time-out, the sender waits M (on average $1/\mu$) time units and then resends the packet which then travels under iid statistical conditions

Packet reaches its destination at time T_i, the process stays in state P till $\mathrm{T_i} + s_i$, then a new packet is transmitted at time $T_i + s_\mathrm{i}$ where $E[s_\mathrm{i}] = 1$

$$\frac{\partial f}{\partial t} = -b\frac{\partial f}{\partial x} + \frac{1}{2}c\frac{\partial^2 f}{\partial x^2} + [\mu W(t) + P(t)]\delta(x - D)$$

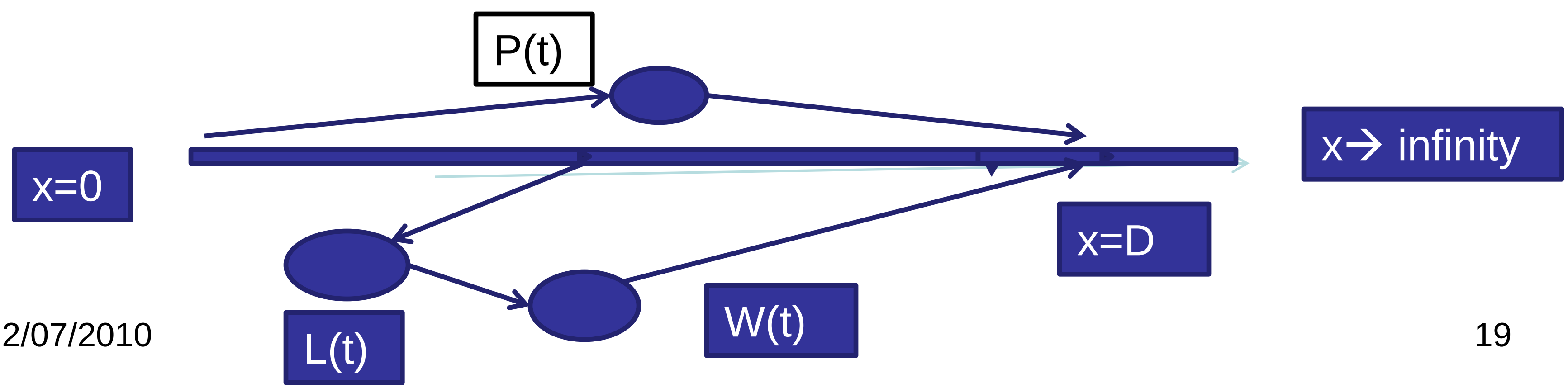
$$\frac{dP(t)}{dt} = -P(t) + \lim_{x \to 0^+}[-bf + \frac{1}{2}c\frac{\partial f}{\partial x}]$$

$$\frac{dL(t)}{dt} = \lambda\int_{0^+}^{\infty} f dx - rL(t)$$

$$\frac{dW(t)}{dt} = r\int_{0^+}^{\infty} f dx + rL(t) - \mu W(t)$$

$$P(t) + L(t) + W(t) + \int_{0^+}^{\infty} f dx = 1; \quad \lim_{x \to 0^+} f = 0.$$

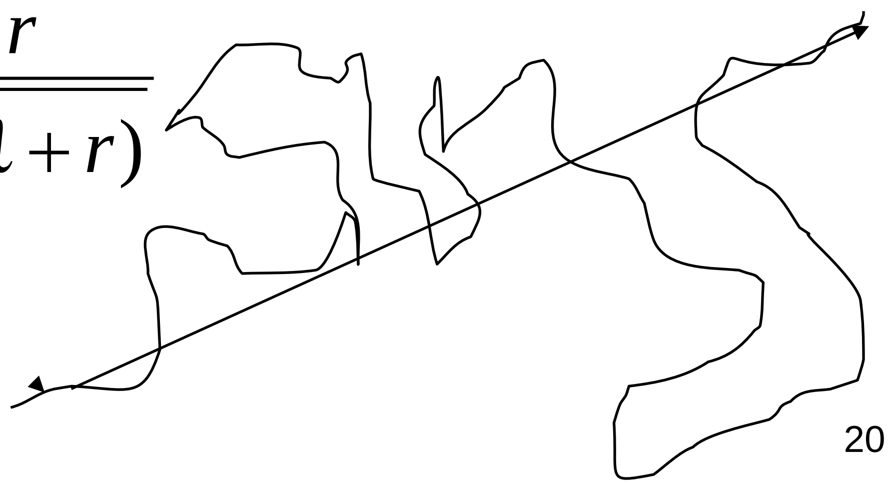
$\mathrm{E[T] = P^{-1}} - 1$ obtained from the stationary solution



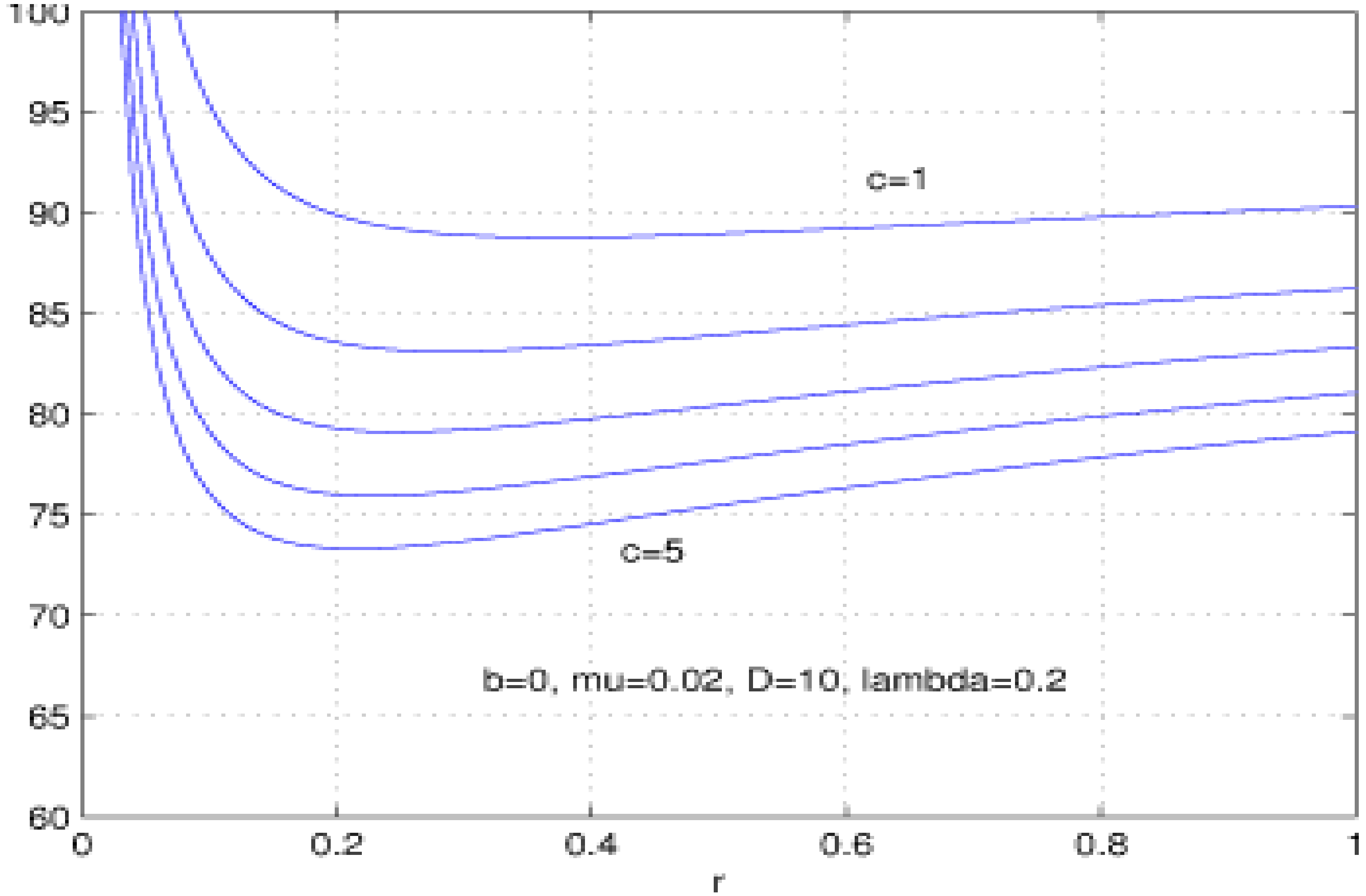
Time to Travel to a Destination conditionally on the Distance D

- Remarkable result: Average Time E[T|D] is Linear in D
- Drift b $\leq$ 0 or b>0, Second Moment Param. c$\geq$0
- Avg Time-Out R=1/r , M=1/μ, then we derive:

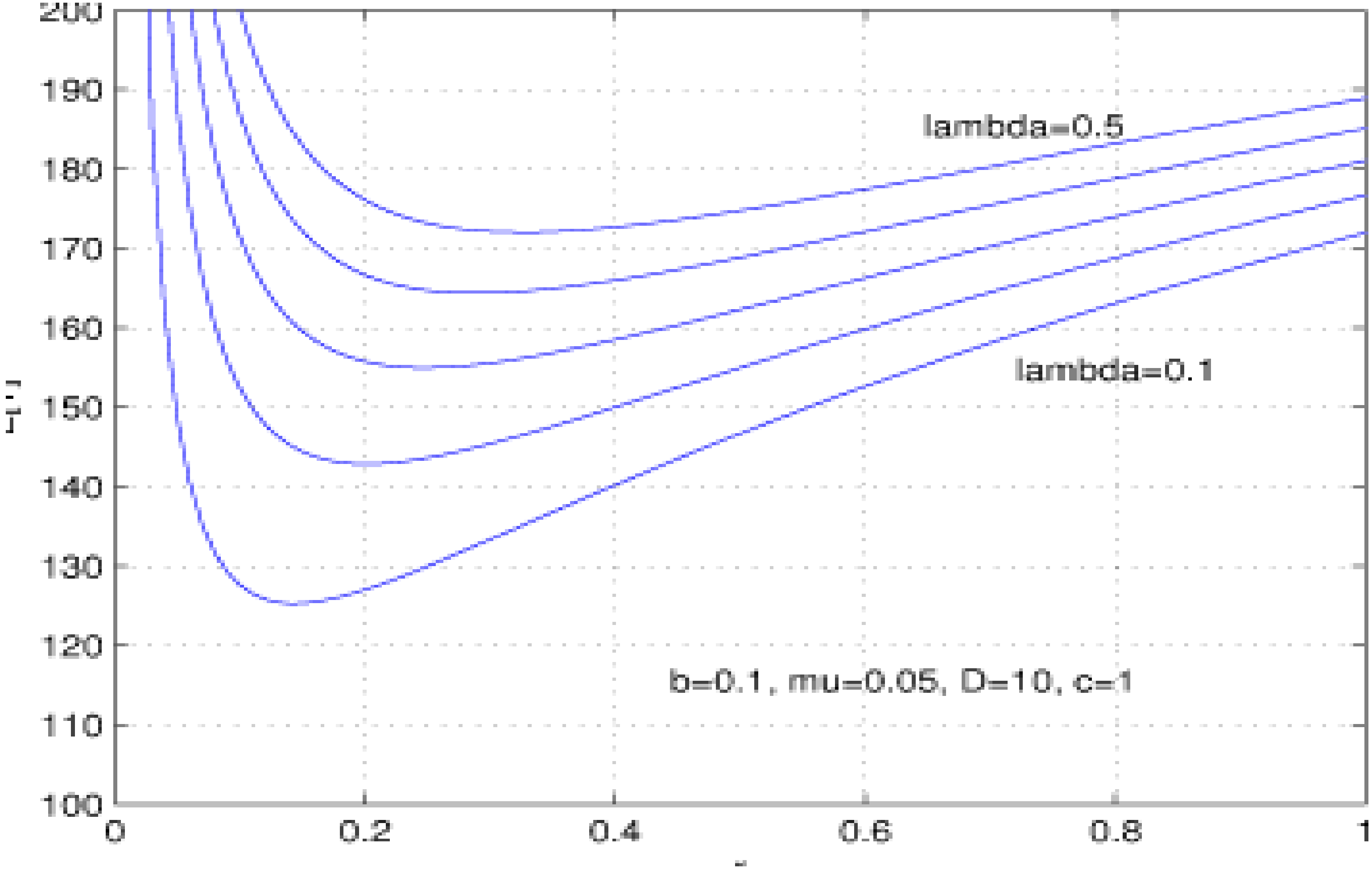
$$E[T \mid D] = -2D \frac{1 + \frac{\lambda + r}{\mu} + \frac{\lambda}{r}}{b - \sqrt{b^2 + 2c(\lambda + r)}}$$



Theory: Average Travel Time for D=10, b=0, 4-Neighbours and M=50 with Losses



Theory: what happens when loss rate increases



Summary of Results for N=1

- Mathematical model assumes exponentially distributed loss time, time-out and re-start time after time-out
- It assumes temporal and spatial homogeneity
- Closed form expressions are obtained for E[T|D] from the diffusion model with jumps for al values of D, b, c, r, λ, μ
- If b<0 (good) E[T|D] is finite
- If b>0 (bad), E[T|D] is finite provided that time-outs (hara-kiri) exist and c>0
- There is a single minimum of E[T] as a function of r
- Other work: Expression for E[T] when the packet chases a moving destination by moving towards the known position of the destination, then chasing it again when it has reached the last known position, etc.
- Sufficient conditions under which the packet gets to the destination when the destination moves

$$\frac{\partial f_i}{\partial t} = -b\frac{\partial f_i}{\partial x_i} + \frac{1}{2}c\frac{\partial^2 f_i}{\partial {x_i}^2} + [\mu W_i(t) + P_i(t)]\delta(x_i - D)$$

$$\frac{dP_i(t)}{dt} = -P_i(t) + \sum_{i=1}^{N} \lim_{x_i \to 0^+} [-bf_i + \frac{1}{2}c\frac{\partial f_i}{\partial x_i}]$$

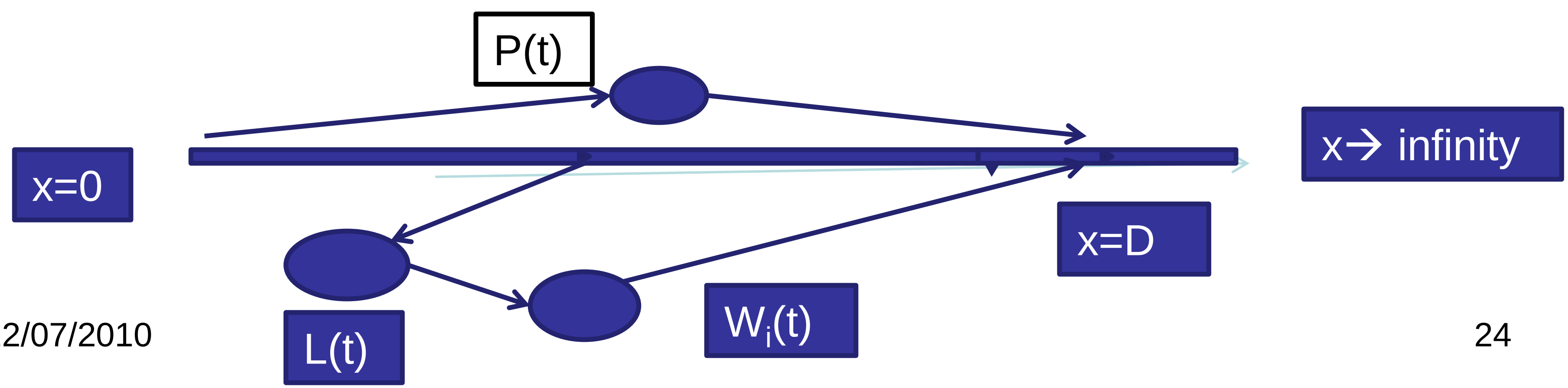
$$\frac{dL_i(t)}{dt} = \lambda \int_{0^+}^{\infty} f_i dx_i - (r + a_i)L_i(t)$$

$$\frac{dW_i(t)}{dt} = r\int_{0^+}^{\infty} f_i dx_i + rL_i(t) - (\mu + a_i)W_i(t)$$

$$a_j \int_{0^+}^{\infty} f_j dx_j = -\sum_{i=1, i\neq j}^{N} \lim_{x_i \to 0^+} [-bf_i + \frac{1}{2}c\frac{\partial f_i}{\partial x_i}]$$

$$P_i(t) + L_i(t) + W_i(t) + \int_{0^+}^{\infty} f_i dx_i = 1; \quad \lim_{x \to 0^+} f = 0.$$

$$\mathrm{E[T^*] = P_i^{-1} - 1 \text{ obtained from the stationary solution}}$$



Time of first arrival among N to Destination conditioned Distance D

$T^* = \inf \{T_1, \ldots , T_N\}$

- Drift b $\leq$ 0 or b>0, Second Moment Param. c$\geq$0
- Avg Time-Out R=1/r , M=1/μ, then we derive:

$$E[T^* | D] = \frac{\frac{-2D}{N}[1 + \frac{\lambda}{r+a} + \frac{r}{\mu+a} + \frac{r\lambda}{(\mu+a)(r+a)}]}{b - \frac{c(N-1)}{2D} - \sqrt{(b - \frac{c(N-1)}{2D})^2 + 2c(\lambda + r)}}$$

Effective Travel Time and Energy

$T^* = \inf \{T_1, \dots , T_N\}$

- $E[\tau_{eff}|D] = [1+E[T^*|D]\,]\,.\,P[\text{searcher is travelling}]$
- $J(N) \geq N.\, E[T^*|D] \geq N.E[\tau_{eff}\,|D]$

$$J(N) \geq \frac{2D[1+\frac{\lambda}{r+a}+\frac{r}{\mu+a}+\frac{r\lambda}{(\mu+a)(r+a)}]}{(\frac{c(N-1)}{2D}-b)-\sqrt{(b-\frac{c(N-1)}{2D})^2+2c(\lambda+r)}}$$

Effective Travel Time Energy Lower Bound

$T^* = \inf \{T_1, \ldots, T_N\}$

$E[\tau_{eff}|D] = [1+E[T^*|D]] \,.\, P[\text{searcher is travelling}]$

$J(N) \geq N.E[T^*|D] > N.E[\tau_{eff}|D]$

$$E[\tau_{eff} \mid D] = \frac{2D/N}{(\frac{c(N-1)}{2D} - b) - \sqrt{(b - \frac{c(N-1)}{2D})^2 + 2c(\lambda + r)}} \approx O(\frac{1}{N^2})$$

$$E[\tau_{eff} \mid D] = \frac{2D}{-b - \sqrt{b^2 + 2c(\lambda + r)}} \qquad for \qquad N = 1$$

$$J(N) \geq NE[\tau_{eff}] = \frac{2D}{(\frac{c(N-1)}{2D} - b) - \sqrt{(b - \frac{c(N-1)}{2D})^2 + 2c(\lambda + r)}}$$